Appendix A

To express the original system of equations in stationary form we detrend each variable by the knowledge available to each generation in their youth. Starting with the education technology, divide both sides of equation (14) by the amount of knowledge available at time , which results in the following trend stationary equation for effective units of labor,

(A1) .

The superscript denotes variables in their stationary form. For example, the variable expresses human capital in terms of the amount of knowledge available to this particular generation.

To determine the stationary level of education spending per child divide both sides of equation (17) by and use equation (15), .

(A2)

The tax rate is already in stationary form, so we do not use a superscript for this variable. This result also applies to the effective wage rate as we demonstrate shortly.

The next step is to detrend the working generation’s decision problem. The children of period become adults in period , so the detrending variable for adults next period is also . This results in the following stationary Euler equation and period specific budget constraints.

(A3)

(A4)

(A5)

For reference, the trend stationary variables for these equations are: , , , and .

Given the constant returns to scale production function and competitive markets assumption, factor payments are stationary. To see this last result, rewrite equations (6) and (7) as follows:

(A6)

(A7)

Factor payments are now a function of the trend stationary human capital and trend stationary capital per worker, . Since human capital and physical capital per worker both grow at the same rate along the balanced growth path, factor payments are stationary (the ratio is constant along the balanced growth path).

Applying the same process to the social security payment, the social security tax rate, and the market clearing condition we have the following stationary equations.

(A8)

(A9)

(A10) .

We can now use the stationary system of equations above to determine the steady state for the economy. First, assume a constant policy set , where variables without denote steady state values. Also assume that all shocks are zero, for . Finally, normalize effective units of labor to one in the steady state. This last assumption allows the model to nest the standard two period economy with overlapping generations when education spending is completely ineffective. Given these assumptions we can solve for the steady state capital stock for a given value of the return to capital using equation (A7). The steady state capital stock can then be used to find the effective wage in equation (A6) and household savings in equation (A10). The steady state effective wage rate, along with the policy set , gives us the steady state replacement rate in equation (A9) and education spending per child in equation (A2). These values and the policy set can be used to find household consumption during the working period, equation (A4). Finally, the steady state social security payment can be found using equation (A8) and retirement consumption can be found using equation (A5). The free parameters are in the education production function and in the Euler equation. With this steady state we can log-linearize the system above to express the variables in log-deviation form, which we provide in Table 1 and discussed in the body of the paper.

Appendix B

This appendix solves the system of equations in Table 1 for policy rule 1. In this case, we set the policy variables and for all . This implies education spending per child and the replacement rate adjust to any changes in the demographic composition of society.

Start with the Euler equation in deviation form (the equation numbering in this appendix is the same as in the body of the paper):

(31)

Now, use equation (29) to express the left-hand side of the Euler equation as follows:

.

Note that this equation imposes the policy restrictions under the first policy rule. Now eliminate the wage rate, (27) , and collect like terms:

.

Finally eliminate capital, (28) lagged one period , and collect like terms:

.

Now, use equation (30) to express the right-hand side of the Euler equation as follows:

Eliminate the social security payment using equations (24) and (25), updated one period, and collect like terms.

Eliminate factor payments using equations (27) and (28), updated one period, and collect like terms.

Finally, eliminate capital and collect like terms.

Use the Euler equation, equating both sides using the equations above, and solve for current period savings, :

(B1)

where .

We now solve the system for our second state variable, human capital. First substitute equation (23) into equation (22).

Next, eliminate the effective wage rate and collect like terms.

(B2)

Equations (B1) and (B2) result in a block recursive system of equations. To express this system in terms of a VAR model, substitute (B2) into (B1). This gives us equations (32) and (33) in the body of the paper. The partial elasticities (reduced form coefficients) of the system are provided in Table 2 and Table 3.

Appendix C

This appendix provides the proof for proposition 4. First, set for each of the coefficients found in Table 2 and Table 3. This results in the following system of equations governing the two key state variables, savings and human capital ( and ).

(C1)

(C2)

Next, conjecture a demographic shock and assume that the system starts in the steady state at the time of the shock (time period zero).

Given this set of assumptions, the sequence for each of the state variables can be found using equations (C1) and (C2).

Period 0

Period 1

Period 2

Period

Given the sequence for savings, the capital stock per worker sequence is as follows (See Table 1).

Period 0

Period 1

Period 2

Period

Given the capital stock per worker sequence and the sequence for human capital, the factor price sequences are as follows (See Table 1).

Period 0

Period 1

Period 2

Period

This last result demonstrates the complete factor price smoothing effect when under policy rule 1. Thus, proving part (a) of Proposition 4. Before determining the consumption profiles for each generation, we need the social security payment sequence (See Table 1).

Period 0

Period 1

Period 2

Period 3

Period

Given the factor price sequences and the social security payment sequence, the consumption sequences are as follows (See Table 1).

Period 0

Period 1

Period 2

Period 3

Period

Thus, the consumption profile for each generation takes the following form:

Initial Old:

Initial Working Generation (Parents):

Children subject to Demographic Shock:

All Future Generations ():

Thus, under policy rule 1, the demographic shock is borne by the generation subject to the shock when and all other generations are left unaffected. This is part (b) in Proposition 4.

Appendix D

This appendix solves the system of equations in Table 1 for policy rule 2. In this case, we set the policy variables and for all . This implies that education spending per child remains constant for each generation and the education tax rate adjusts to any changes in the demographic composition of society.

From equation (22) and (23) in the body of the paper, we have,

(22’)

(23’)

which demonstrates that the education component drops out of the model. To solve the system for the one remaining state variable, savings, we follow the same procedure as in appendix B. First, start with the left-hand side of the Euler equation,

Eliminate the education tax rate (23’) and collect like terms

Now eliminate the factor payment and collect like terms,

Finally, eliminate capital using the market clearing equation and collecting like terms,

Now solve for savings on the right-hand side of the Euler equation. Again, the solution process is equivalent to the one found in appendix B (minus the education variable):

Equating both sides of Euler equation and solving for savings in period results in the equation of motion for the savings variable (equation (34)). The coefficients are provided in Table 7.